1/31/17 "Quality is not an act, it is a habit" - Aristotle

HW: TBD
Test 1 on Thursday February 16

AIM: How do we determine max/min values?

Warm Up:

1) Find 2 positive numbers whose sum is 40 and whose product is 175.

Let: x = 1st Positive #
y = 2nd Positive #

$x + y = 40$
$x = 40 - y$

$xy = 175$

$(40 - y)y = 175$

$40y - y^2 = 175$

$0 = y^2 - 40y + 175$

$(y - 35)(y - 5)$

$y = 35 \quad y = 5$

Solve for x

$x + 5 = 40$
$x = 35$
$(35, 5)$

$x + 35 = 40$
$x = 5$
$(5, 35)$

5 and 35

Optimization Problems - Classwork

Many times in life we are asked to do an optimization problem - that is, find the largest or smallest value of some quantity that will fufill a need. Typical situations are:

- find the route which will minimize the time it takes me to get to school.
- build a structure using the least amount of material.
- build a structure costing the least amount of money.
- build a yard enclosing the most amount of space.
- find the least medication one should take to help a medical problem.
- find how the most one should charge for a CD in order to make as much money as possible.

All of these situations have something in common - they are all trying to maximize or minimize some quantity. This lends itself to a calculus solution. We have spent the better part of last month trying to find maximum and minimum values of functions. In every optimization problem, you are always looking for a quantity to be maximimed or minimized. So in solving word problems, you must look carefully for certain words among all the verbiage. Look for words like "minimize area", "smallest volume", "least amount of time", "shortest distance", "cheapest price." On the following pages, there are a wealth of problems. Quickly examine each and underline the key words which tell you what kind of problem it is.

Methods for Solving Optimization Problems

1. Assign variables to all given quantities and quantities to be determined. Don't be afraid to use letters you usually do not use (p,m,g, etc.). When feasible, make a sketch of the problem.

2. Making a chart of possible answers allows you to see a relationship between variables. While not necessary, it is helpful.

3. Write a "primary" equation for the quantity you found that needs to be maximized or minimized

   Area of Rectangle = length • width — Hypotenuse = $\sqrt{x^2+y^2}$
   Distance = rate • time — Perimeter of rectangle = 2• length + 2• width
   Volume of rectangular solid = length • width • height — Volume of cylinder = π(radius^2) • height

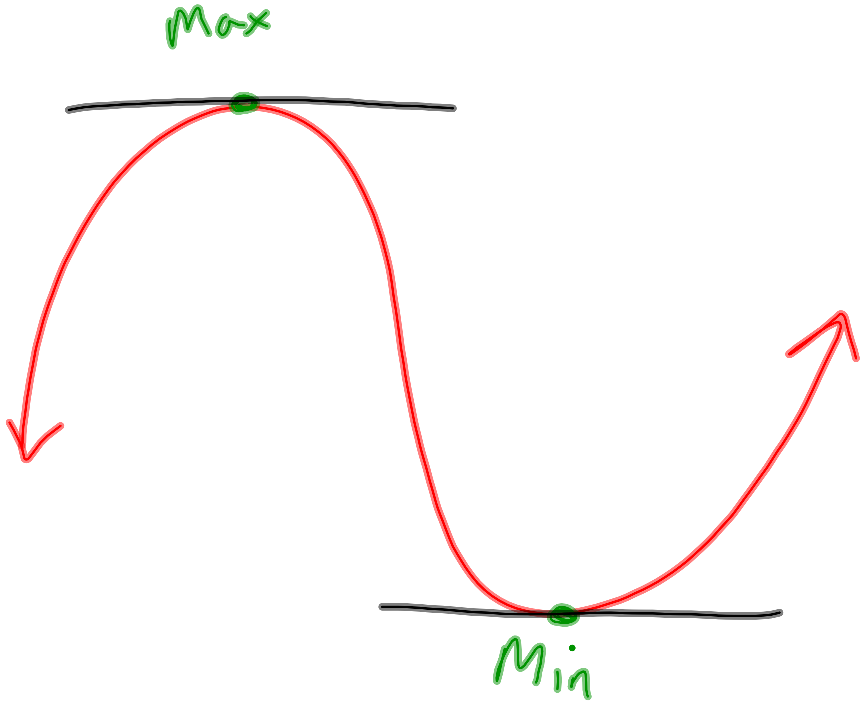
4. Reduce the right side of this "primary equation" to one having a single variable. If there is more than one variable on the right side, you must write a "secondary" equation (a restriction or constraint) relating the variables of the primary equation.

5. Take the derivative of the equation and **set equal to zero.** If you get more than one answer, make a sign chart to determine whether it represents a maximum or minimum. Pay attention to whether that value makes sense. Time is rarely negative (it can't take negative 7 hours to run a race). You cannot use more than you have (you can't have a length of 8 feet when you only have 6 feet of fencing).

6. Be sure that you answer the question that is asked. If you are asked to find a minimum or maximum value of some quantity, you must plug your answer from (4) into your primary equation.

7. If you are to find a maximum or minimum on a closed interval, you must test the endpoints as well. Make sure your work is clear.

8. You can verify your answers by graphing your primary equation with one variable on the calculator. Use your $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ maximum or minimum function.


Max
Min

Example 1) Two numbers add up to 40. Find the numbers that maximize their product.

Let $x =$

$y =$

Smaller Number	1	10	15	. . .	.	20
Larger Number	39	30	25	. .	.	20
Product	39	300	375			400

Primary *Secondary*

$xy = \text{Product}$

$x + y = 40$

$x = 40 - y$

$(40-y)(y) = \text{Product}$

$40y - y^2 = P$

$40 - 2y = P'$

$40 - 2y = 0$

$40 = 2y$

$y = 20$

$x = 20$

$x + y = 40$

$x + 20 = 40$

$x = 20$

P' $\oplus$ 20 $\ominus$

Test 10 Test 30

Verifies $y = 20$ is a max

20, 20

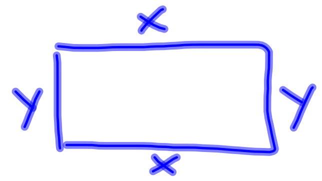
72

$36 > x > 0$

Example 2) A rectangle has a perimeter of [redacted] feet. What length and width should it have so that its area is a maximum? What is this maximum area?

Let $x =$
$y =$

Width						
Length						
Area						



Primary

$$A = xy$$

$$A = (36-y)(y)$$

$$A = 36y - y^2$$

$$A' = 36 - 2y$$

$$0 = 36 - 2y$$

$$2y = 36$$

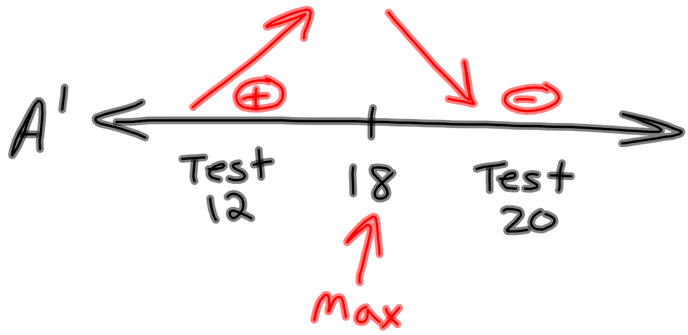
$$y = 18$$

Secondary

$$2x + 2y = 72$$

$$x + y = 36$$

$$x = 36 - y$$



Find x:

$$2(18) + 2x = 72$$

$$36 + 2x = 72$$

$$2x = 36$$

$$x = 18$$

Length $= \boxed{18\text{ ft}}$

Width $= \boxed{18\text{ ft}}$

Max Area $= 18 \cdot 18 = \boxed{324\text{ ft}^2}$

* If we have a closed interval (restrictions), be sure to check the endpoints as well.

$x > 0$
$y > 0$

Example 3) Find two positive numbers that $\boxed{\text{minimize the sum of twice the first number plus the second}}$ if the product of the two numbers is 288.

Let $x =$
$y =$

First Number						
Second Number						
Sum						

Primary

$$2x + y = \text{Sum}$$

$$2\left(\frac{288}{y}\right) + y = S$$

$$\frac{576}{y} + y = S$$

$$576y^{-1} + y = S$$

$$S' = -576y^{-2} + 1$$

$$0 = -576y^{-2} + 1$$

$$0 = -\frac{576}{y^2} + 1$$

$$\frac{576}{y^2} = 1$$

$$y^2 = 576$$

$$y = \pm 24$$

$$y = 24$$

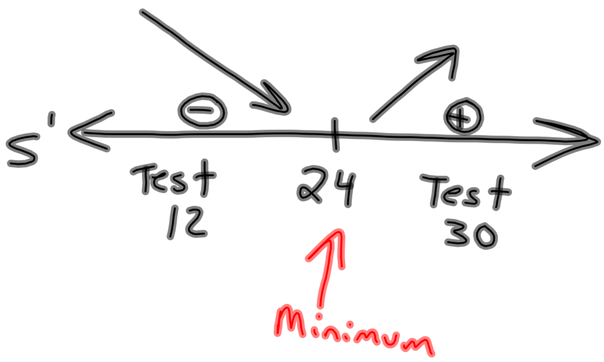
b/c $y > 0$

Secondary

$$xy = 288$$

$$x = \frac{288}{y}$$

S' DNE when $y = 0$
reject b/c $y > 0$



To find x

$$24 \cdot x = 288$$

$$x = 12$$

Numbers are 12 and 24

STEPS for Max/Min

1) Identify the function to be Max/Min
   (set restrictions)

2) Find Derivative of that function
   (You may need a second equation if there are multiple variables)

3) Set Derivative = 0 and solve
   (Find values that make it undefined if needed)

4) Use First Derivative Test (Number Line) to verify Max/Min

5) Answer the question asked.

HW Page 111 #1

Primary:

$x^2 + y^2 = \text{Minimum}$

$x^2 + (10-x)^2 = \text{Min}$

$x^2 + x^2 - 20x + 100 = \text{min}$

$2x^2 - 20x + 100 = \text{min}$

$4x - 20 = 0$

$4x = 20$

$x = 5$

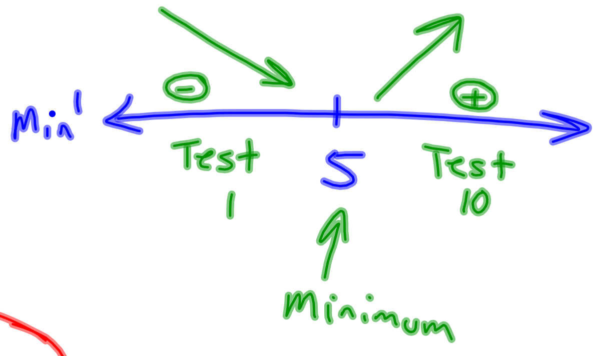
Secondary

$x + y = 10$

$y = 10 - x$

$5 + y = 10$

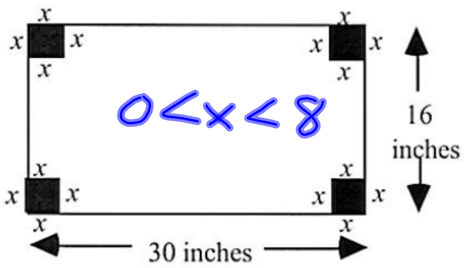
$y = 5$



Numbers 5, 5

Example 4) An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut out to create a box with greatest volume. What is the maximum volume as well?

Primary



$V = L \cdot W \cdot H$

$V = (30-2x)(16-2x)(x)$

$V = 480x - 92x^2 + 4x^3$

$V' = 480 - 184x + 12x^2$

$0 = 480 - 184x + 12x^2$

$0 = 4(120 - 46x + 3x^2)$

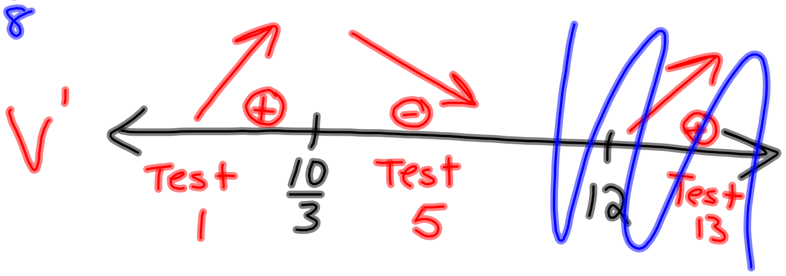
Quad Formula

$$x = \frac{46 \pm \sqrt{(-46)^2 - 4(3)(120)}}{2(3)}$$

$x = \cancel{12}, \frac{10}{3}$

reject

$0<x<8$



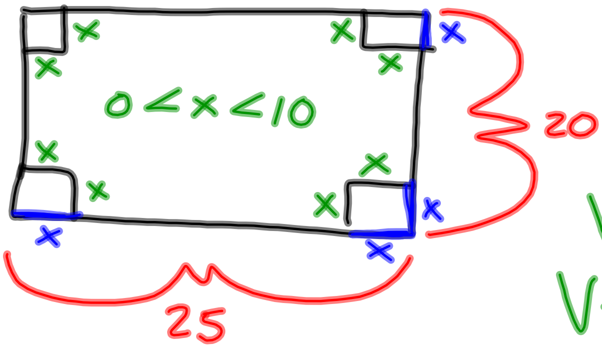
$$\boxed{\text{Size of square is } \tfrac{10}{3} \text{ in}}$$

Max Volume:

Volume: $480\left(\frac{10}{3}\right) - 92\left(\frac{10}{3}\right)^2 + 4\left(\frac{10}{3}\right)^3$

$V \approx \boxed{725.926 \text{ in}^3}$

HW: Page 122 #2



Max:

$V = L \cdot W \cdot H$

$V = (20-2x)(25-2x)x$

$V = (500 - 40x - 50x + 4x^2)x$

$V = 500x - 90x^2 + 4x^3$

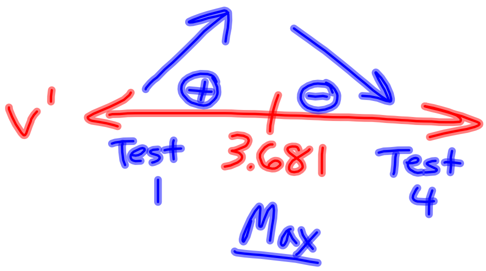
$V' = 500 - 180x + 12x^2$

$$0 = 500 - 180x + 12x^2$$
$$= 12x^2 - 180x + 500$$
$$= 4(3x^2 - 45x + 125)$$

$$x = \frac{45 \pm \sqrt{(-45)^2 - 4(3)(125)}}{2(3)}$$

$x =$ 11.319 and 3.681

reject



The squares should have sides of 3.681 in

The Volume is : $500(3.681) - 90(3.681)^2 + 4(3.681)^3$

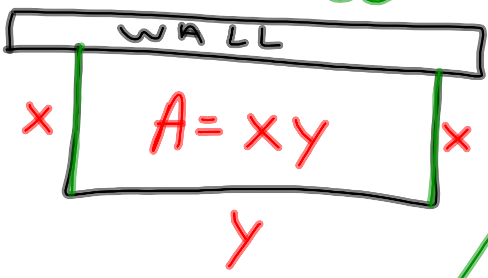
$$\boxed{V = 820.528 \text{ in}^3}$$

Page 125

1. A farmer has 600 m of fencing with which he plans to enclose a rectangular pen adjacent to a long existing wall. He will use the wall for one side of the pen and the available fencing for the three remaining sides. What is the maximum area that he can enclose this way?

$0 < y < 600$

$0 < x < 300$



$2x + y = 600$

$y = 600 - 2x$

Max:

$A = xy$

$A = x(600 - 2x)$

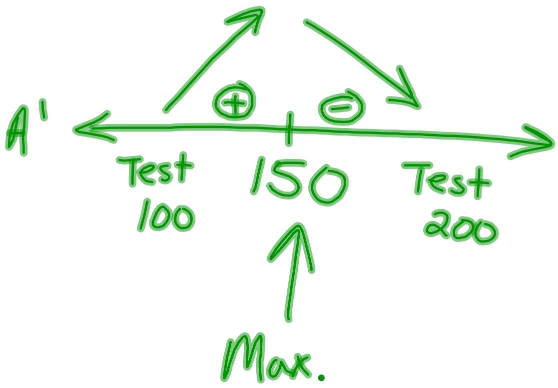
$A = 600x - 2x^2$

$A' = 600 - 4x$

$0 = 600 - 4x$

$4x = 600$

$x = 150$



Find y:

$2x + y = 600$

$2(150) + y = 600$

$y = 300$

Max Area:

$A = xy$

$A = 150(300)$

$\boxed{A = 45{,}000\ m^2}$

HW: Page 111 #4